

# Antiderivatives

A function  $F$  is **an antiderivative** of  $f$  on an interval when  $F'(x)=f(x)$  for all  $x$  in the interval.

$$F(x) = x^3 \quad \frac{d}{dx} F(x) = 3x^2$$



$$x^3$$

$$x^3 + 25$$

$$x^3 - 3$$



$$G(x) = x^2 + C$$

*constant of integration*

*family of antiderivatives*

**Differential Equations:** involve  $x$ ,  $y$ , and the derivatives of  $y$ .

$$y' = 3x$$

$$y' = x^2 + 1$$

Find the general solution of the differential equation

$$y'=2$$

$$y = 2x + C$$

$$\frac{dy}{dx} = f(x)$$

$$dy = f(x)dx$$

- Antidifferentiation  
- indefinite integral

$$\int f(x) dx = F(x) + C$$

*Variable of integration* (with arrow pointing to  $dx$ )

*integral* (with arrow pointing to  $\int$ )

*antiderivative* (with arrow pointing to  $F(x)$ )

*constant of integr.* (with arrow pointing to  $C$ )

$\int 5 dx$   
 $5x + C$

Rules

$$\int k \, dx$$
$$kx + C$$

$$\int 3x \, dx$$

$$3 \int x \, dx$$

$$3 \left( \frac{x^2}{2} \right) + C$$

$$\frac{3}{2} x^2 + C$$



$$\int \frac{1}{x^3} = \int x^{-3}$$
$$\frac{x^{-2}}{-2} + C$$
$$-\frac{1}{2x^2} + C$$

$$\int \sqrt{x}$$
$$\int x^{\frac{1}{2}} = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2\sqrt{x^3}}{3} + C$$

$$\int 2 \sin x \, dx$$
$$-2 \cos x + C$$

$$\int (3x^4 - 5x^2 + x) dx$$

$$\frac{3x^5}{5} - \frac{5x^3}{3} + \frac{x^2}{2} + C$$

$$\int \frac{x+1}{\sqrt{x}} dx$$
$$\int (x+1)(x^{-1/2})$$
$$\int x^{1/2} + x^{-1/2}$$
$$\frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2}$$
$$\frac{2}{3}x^{3/2} + 2x^{1/2} + C$$

$$\int \frac{\sin x}{\cos^2 x} dx$$

$$\int \frac{1}{\cos x} \frac{\sin x}{\cos x} dx$$

$$\int \sec x \tan x dx$$

$$\sec x + C$$